MINISTRY OF EDUCATION AND TRAINING
HO CHI MINH CITY UNIVERSITY OF TECHNOLOGY AND ENGINEERING

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RESEARCH AND DEVELOPMENT OF COMPLIANT CONSTANT-TORQUE MECHANISMS IN BIOMECHANICAL APPLICATIONS

Major: Mechanical Engineering
Major code: 9520103

DOCTORAL DISSERTATION SUMMARY

HO CHI MINH CITY – 2026

This dissertation was completed at **Ho Chi Minh City University of Technology and Engineering**

Supervisor 1: Assoc. Prof. Dr. PHAM HUY TUAN
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Reviewer 1:
Reviewer 2:
Reviewer 3:

The dissertation will be defended before the Institutional-Level Dissertation Evaluation Committee at Ho Chi Minh City University of Technology and Engineering
on ……, ……, 2026

LIST OF PUBLICATIONS

I. Publications Arising from the Dissertation

[1] **T.-V. Phan**, V. M. Truong, H.-T. Pham, V.-K. Nguyen, and A. Bukayeva, "Robust design and optimization of a large-stroke compliant constant-torque mechanism under fabrication uncertainties," *Mechanism and Machine Theory,* vol. 215, p. 106179, 2025/11/01/ 2025, doi: https://doi.org/10.1016/j.mechmachtheory.2025.106179. **(SCIE, Q1, IF 5.3)**

[2] **T.-V. Phan**, V. M. Truong, H.-T. Pham, and V.-K. Nguyen, "Design of a Novel Large-Stroke Compliant Constant-Torque Mechanism Based on Chained Beam-Constraint Model," *Journal of Mechanisms and Robotics,* vol. 16, no. 8, 2023, doi: 10.1115/1.4063980. **(SCIE, Q1, IF 3.2)**

[3]. **T.-V. Phan** and H.-T. Pham, "Design and Optimization of a Large-Stroke Compliant Constant-Torque Mechanism," *Journal of Technical Education Science,* vol. 68, pp. 93-100, 2022. https://doi.org/10.54644/jte.68.2022.1098

[4]. **T.-V. Phan**, H.-T. Pham, and C.-N. Truong, "Design and Analysis of a Compliant Constant-Torque Mechanism for Rehabilitation Devices," Cham, 2020: Springer International Publishing, in Advanced Materials, pp. 541-549 **(Scopus)**

II. Other Publications Related to the Research Direction

[1]. D. C. Nguyen, **T. V. Phan**, and H. T. Pham, "Design and Analysis of a Compliant Gripper Integrated with Constant-Force and Static Balanced Mechanism for Micro Manipulation," in *2018 4th International Conference on Green Technology and Sustainable Development (GTSD)*, 23-24 Nov. 2018 2018, pp. 291-295, doi: 10.1109/GTSD.2018.8595638 **(IEEE)**

[2]. H.-T. Pham, **T.-V. Phan**, and V.-T. Mai, "Optimization design of a carbon fibre prosthetic foot for amputee," *Acta Scientific Orthopaedics,* vol. 3, no. 10, pp. 16-21, 2020.

[3] H.-T. Pham, V.-K. Nguyen, Q.-K. Dang, T. V. A. Duong, D.-T. Nguyen, and **T.-V. Phan**, "Design Optimization of Compliant Mechanisms for Vibration- Assisted Machining Applications Using a Hybrid Six Sigma, RSM-FEM, and NSGA-II Approach," *Journal of Machine Engineering,* vol. 23, no. 2, pp. 135-158, 2023 2023, doi: 10.36897/jme/166500. **(Scopus, Q2)**

III. Intellectual Property

[1] Patent application entitled *"Knee Joint Functional Assist Device Integrating a Compliant Constant-Torque Mechanism,"*
National Office of Intellectual Property – Ministry of Science and Technology, Vietnam
(Application accepted on November 20, 2025)

INTRODUCTION

1. **Rationale of the study**

A Constant-Torque Mechanism (CTM) is a mechanism capable of generating an approximately constant torque over a specified range of input rotation. Owing to this characteristic, CTMs provide an effective solution for devices requiring torque regulation without reliance on complex sensing or control systems, thereby simplifying structural design and reducing system cost. When a CTM is realized in the form of a compliant mechanism, it is referred to as a Compliant Constant-Torque Mechanism (CCTM). By combining the advantages of compliant mechanisms with the ability to maintain constant torque, CCTMs have emerged as a promising research direction in the field of biomechanics.

However, one of the major limitations of compliant mechanisms in general, and CCTMs in particular, is their restricted operating range, as they must function within the elastic limit of the material. This constraint significantly reduces the practical applicability of CCTMs. Therefore, the research and development of CCTM designs with a large working range, while simultaneously ensuring allowable stress levels and a compact structure, has become an urgent requirement.

On the other hand, most existing CCTMs with a large constant-torque range are manufactured using CNC milling processes from polymer materials. During machining, cutting forces may induce geometric deviations in the compliant beams, thereby affecting the operational performance and efficiency of the mechanism. To date, however, no comprehensive study has systematically investigated the influence of manufacturing-induced uncertainties on the actual performance of CCTMs within an optimal design framework. Consequently, in addition to developing design solutions that extend the working range of CCTMs, it is essential to consider random factors arising during the manufacturing process to ensure that the fabricated mechanisms still meet the intended design requirements.

2. **Research objectives**

General objective

The overall objective of this study is to investigate the design and optimization of CCTMs with a large constant-torque range and compact size, suitable for application in biomechanical devices. The proposed design solutions must ensure that the desired performance characteristics are maintained even in the presence of random manufacturing-induced uncertainties, through a reliability-based optimization approach.

3. **Research tasks**

To achieve the aforementioned objectives, this dissertation addresses the following research tasks:

- Reviewing and systematically summarizing relevant studies.
- Proposing new CCTM designs.
- Developing computational models based on the Chained Beam Constraint Model (CBCM).
- Performing reliability analysis of CCTMs using the First-Order Reliability Method (FORM).
- Integrating the CBCM-based computational model and FORM-based reliability analysis into the multi-objective optimization algorithm NSGA-II.
- Conducting reliability-based optimal design of CCTMs.
- Fabricating CCTM prototypes and conducting experimental measurements of torque and displacement.
- Integrating the optimized CCTM into at least one biomechanical application.

4. **Scope of the study**

Research objects

- Compliant constant-torque mechanism structures exhibiting large deformation, with bending of slender beams as the primary deformation mode.
- Priority is given to structures that can be integrated into biomechanical devices, featuring compact geometry and appropriate angular operating ranges.

Methods and tools

- Static and kinetostatic analyses based on CBCM to model large deformations.
- Reliability analysis using the FORM approach.
- Multi-objective optimization using Non-dominated Sorting Genetic Algorithm II (NSGA-II) with reliability constraints.
- Design, simulation, and fabrication of experimental prototypes for model validation.

Materials and operating conditions

Assumptions and limitations

Exclusions

5. **Research approach and methodology**

This dissertation builds upon existing studies to select relevant concepts, develop design ideas, and perform optimization toward the defined objectives. The research is conducted progressively through several stages: initially, the optimal design of a novel CCTM is developed; subsequently, the design is extended to achieve a large working range; finally, a large-range CCTM incorporating random manufacturing uncertainties is investigated.

To achieve these objectives, both analytical and numerical approaches are employed in parallel. The analytical approach is based on the CBCM, while the numerical approach relies on finite element analysis using ABAQUS, combined with optimization using the NSGA-II implemented in MATLAB. Experimental validation is ultimately conducted to verify the simulation results.

6. Scientific and practical significance of the study

Scientific significance

This dissertation proposes novel CCTM designs with larger working ranges and more compact configurations compared to previous studies.

Static and kinetostatic analyses of the proposed CCTMs are performed using the CBCM and integrated into the multi-objective genetic algorithm NSGA-II.

FORM-based reliability analysis, CBCM-based modeling, and NSGA-II optimization are combined to formulate a reliability-based multi-objective optimal design problem for CCTMs. This optimization framework accounts for random manufacturing-induced uncertainties, ensuring that the fabricated CCTMs satisfy the design objectives.

A method for evaluating geometric and dimensional errors of arbitrarily curved slender beams after machining is proposed. This method enables the assessment of machining accuracy for compliant mechanisms with complex geometries that cannot be inspected using conventional mechanical measurement tools.

Practical significance

This dissertation proposes a biomechanical application of CCTMs: a knee joint assistive device integrating a CCTM, which helps individuals with impaired knee function perform daily walking activities more easily.

The research outcomes serve as a valuable reference for future studies in the field of compliant mechanisms in general, and compliant constant-torque mechanisms in particular.

7. Structure of the dissertation

The dissertation consists of an Introduction and six Chapters.

CHAPTER 1 OVERVIEW

1.1 Compliant mechanisms

1.1.1 General concept

Compliant mechanisms, also referred to as flexible mechanisms, enable the transmission or transformation of motion, force, and torque through the elastic deformation of compliant elements.

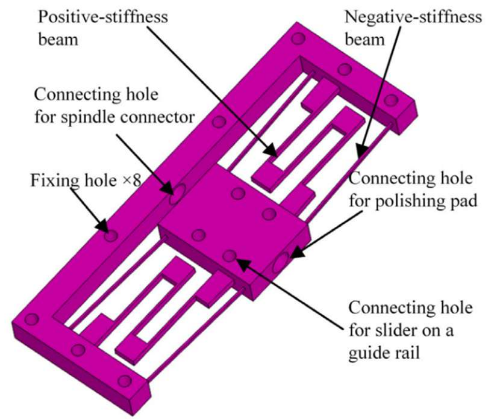


Fig 1.1 Compliant constant-force mechanism for polishing robots [2]

1.1.2 Applications of compliant mechanisms

Owing to their inherent characteristics, compliant mechanisms have attracted increasing attention and have been widely applied in various fields. In addition, from a functional perspective, compliant mechanisms can be classified into constant-force mechanisms, constant-torque mechanisms, bistable mechanisms, and multistable mechanisms.

1.2 Compliant constant-torque mechanisms

1.2.1 Introduction

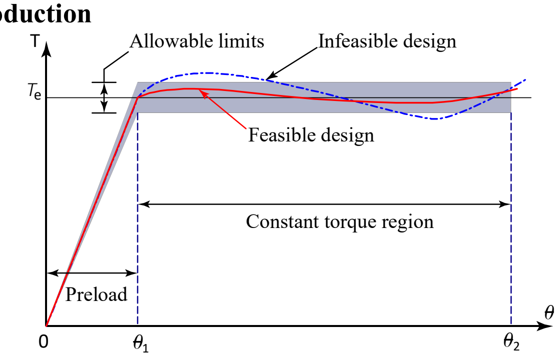


Fig 1.4 Torque – Angle curve of constant-torque mechanisms

1.3 Review and assessment of compliant constant-torque mechanisms

In the research direction of optimal design of CCTMs, several issues can be identified as follows:

- The optimization of CCTMs employing curved beams remains complex, as it increases the number of design variables, leading to high computational complexity and cost in the optimization process.
- The overall size of existing CCTMs is still relatively large, while the constant-torque operating range remains limited, thereby reducing their practical applicability.
- Some CCTMs exhibit complex structural configurations that require assembly, which compromises one of the key advantages of compliant mechanisms, namely monolithic construction.
- **To date, no studies have integrated the CBCM into optimization algorithms** to fully exploit its computational efficiency in reducing the cost of optimal design problems.
- **Furthermore, no studies have considered the influence of random factors that alter the optimized geometry and dimensions**, potentially causing the mechanism to fail to meet design objectives or, in worse cases, lose its constant-torque characteristic.

Therefore, the research direction of this dissertation aims to address the aforementioned issues by exploring new CCTM designs with simpler structures, more compact configurations, and monolithic fabrication, while achieving a large constant-torque operating range (greater than 65°) to enhance practical applicability. In addition, reliability-based optimization is conducted to account for random variables and uncertainties, thereby ensuring high design reliability.

CHAPTER 2 THEORETICAL BACKGROUND

2.1 Bézier curves

A Bézier curve is a parametric approximation curve that is controlled by a set of control points.

2.2 Genetic algorithms (GA) and NSGA-II

Genetic algorithms (GA) are commonly used to solve single-objective optimization problems, while the Non-dominated Sorting Genetic Algorithm II (NSGA-II) is employed for multi-objective optimization problems.

2.3 Finite element method

The finite element method (FEM) implemented in the commercial software ABAQUS is employed and integrated with MATLAB for numerical analysis and optimization.

2.4 Chained beam-constraint-model (CBCM)

In the CBCM approach, a beam is discretized into N elements, where each element is modeled using the Beam Constraint Model (BCM) [85]. The global force and displacement parameters are then obtained by combining the element models with static equilibrium equations and geometric constraint conditions.

2.5 Reliability-based optimization

Considering random variations in design variables or model parameters as additional constraints in the optimization problem ensures that the obtained optimal solutions remain well within the safe domain. This class of problems is referred to as reliability-based design optimization (RBDO).

2.6 First-order reliability method (FORM)

The First-Order Reliability Method (FORM) determines the shortest distance from the origin O in the standardized normal space (U-space) to the limit-state function. This distance is defined through the design point located on the limit-state surface.

2.7 PEEK material

Polyether ether ketone (PEEK) is a thermoplastic polymer characterized by a high yield stress-to-elastic modulus ratio (σ_y/E), making it particularly suitable for large-deformation mechanisms such as CCTMs.

CHAPTER 3 DESIGN AND OPTIMIZATION OF COMPLIANT CONSTANT-TORQUE MECHANISMS BASED ON GENETIC ALGORITHMS

3.1 Optimization of CCTMs using GA and FEM

In this section, the proposed approach is applied to two specific case studies to optimally design two compliant constant-torque mechanisms, referred to as CCTM-1 and CCTM-2.

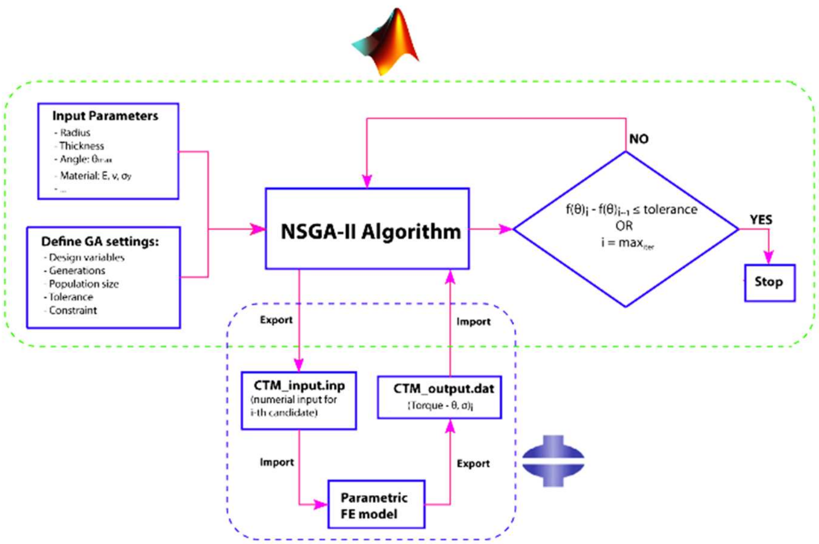
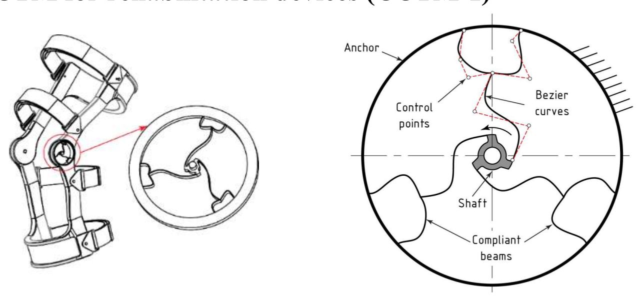


Fig 3.1 Flowchart of the optimization algorithm using genetic algorithm

3.1.1 CCTM for rehabilitation devices (CCTM-1)



a) b)

Fig 3.2 a) Model of a knee joint rehabilitation device, b) design concept of CCTM-1

3.1.1.1 Design of CCTM-1

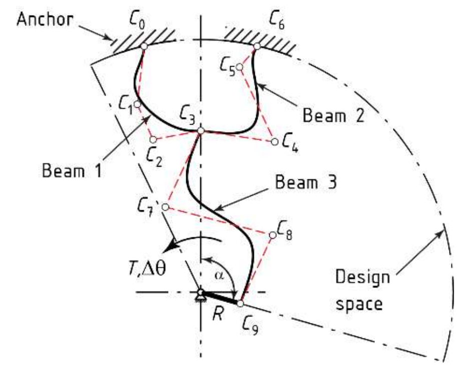


Fig 3.3 Schematic of the design variables and boundary conditions of one branch of CCTM-1

3.1.1.2 Optimization of CCTM-1

Table 3.1 Optimization formulation of CCTM-1

1.*Objective*:
– Minimize the variation of the torque follow Eq. (3.1)
2.*Design variables*:
– Control points: $C_i(x, y)$ $(i = 0 \div 9)$
– In-plane thickness: *w*
3.Constraints:
i. g_1: $C_i(x) < 0$ $(i = 0 \div 2; 7)$; $C_i(x) > 0$ $(i = 4 \div 6; 8)$
ii. $g_2: 0.7 \leq w \leq 1.5$ (mm)
iii. The maximum stress, g_3: $\sigma_m < \sigma_y/SF$

The optimization objective is to minimize torque variation over the operating range of the mechanism, regardless of changes in the input rotation angle, with the objective function defined in Eq. (3.1).

$$Min f(\theta) = \int_a^b (T - T_e)^2 \, d\theta \quad (3.1)$$

3.1.1.3 Results of CCTM-1

Table 3.2 Optimal values of the design variables (DVs) of CCTM-1

DVs	Value (mm)	DVs	Value (mm)
$C_0(x, y)$	(-20.69, 89.76)	$C_5(x, y)$	(21.99, 68.23)
$C_1(x, y)$	(-20.56, 73.49)	$C_6(x, y)$	(20.80, 89.64)
$C_2(x, y)$	(-13.68, 71.76)	$C_7(x, y)$	(-16.96, 41.85)
$C_3(x, y)$	(0.00, 77.37)	$C_8(x, y)$	(21.74, 40.95)

$C_4(x,y)$	(21.80, 68.00)	$C_9(x,y)$	(14.43, -4.10)
w	0.90		

The out-of-plane thickness of the mechanism is set to t = 10 mm.

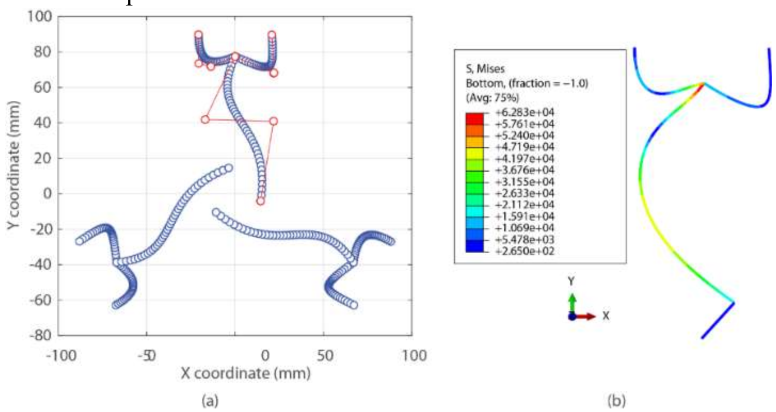


Fig 3.4 Optimal Bézier curves of CCTM-1 (a) and simulation results from ABAQUS (b)

The results show good agreement between the 1D and 3D models. The constant-torque characteristic is achieved over approximately two-thirds of the operating range, from 20° to over 60°, with torque deviation not exceeding 5% across the entire input rotation range.

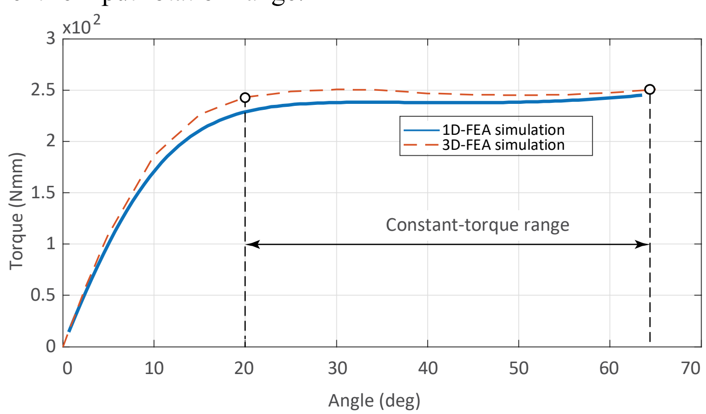


Fig 3.6 Torque – Angle curve of the CCTM-1

3.1.2 Two-Stage Compliant Constant-Torque Mechanism (CCTM-2)

3.1.2.1 Design of CCTM-2

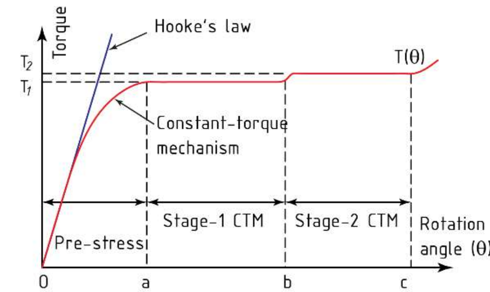


Fig 3.7 Torque – Angle curve of a stacked CCTM

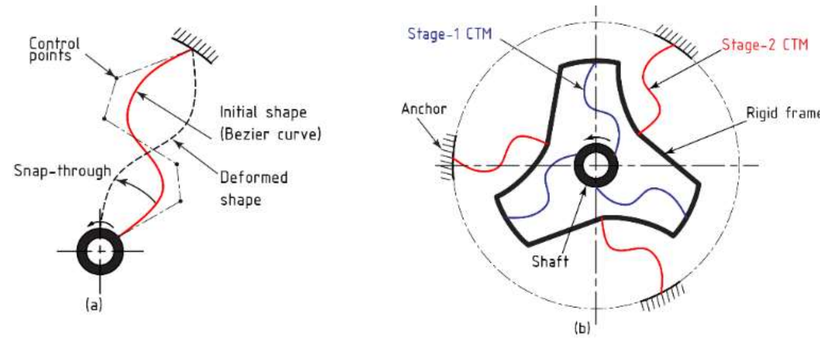


Fig 3.8 Schematic design of a CTM (a) and its 2-stages long-stroke design concept (b)

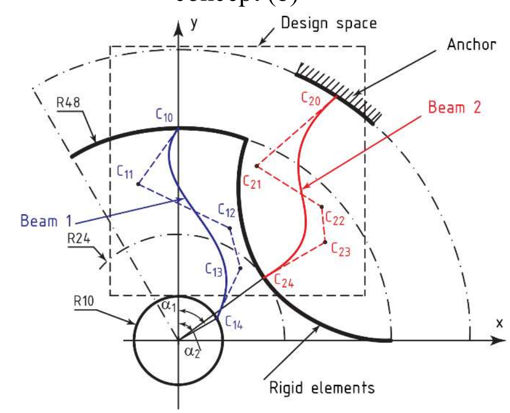


Fig 3.9 Schematic of the design variables of CCTM-2

3.1.2.2 Optimization of CCTM-2

$$\mathrm{Min}[f(\theta)] = \int_a^b (T - T_1)^2\, d\theta + \int_b^c (T - T_2)^2\, d\theta \tag{3.2}$$

Table 3.3 Formulation of the CCTM-2 optimization

1.*Objective*:

Minimize the variation of the torque follow Eq. (3.2)

2. *Design variables*:

– Control points: $C_{ij}(x,y)$ $(i = 1 \div 2, j = 0 \div 4)$

– In-plane thickness: w

– Angles: α_1, α_2

3.Constraints:

i. g_1: $C_{11}(x) < 0$;

ii. g_2: $C_{1j}(x) > C_{2j}(x)$ $(j = 0 \div 4)$

iii. g_3: $0.7 \le w \le 1.5$ (mm)

iv. g_4: $\pi/18 \le \alpha \le \pi/1.5$ (rad)

v. The maximum stress, g_5: $\sigma_m < \sigma_y/SF$

3.1.2.3 Results of CCTM-2

Table 3.4 Optimal design variables of the CCTM-2

DVs	Value (mm)	DVs	Value (mm)
$C_{10}(x,y)$	(0.00, 48.00)	$C_{22}(x,y)$	(12.70, 35.60)
$C_{11}(x,y)$	(-0.90, 25.47)	$C_{23}(x,y)$	(43.03, 15.90)
$C_{12}(x,y)$	(1.00, 32.53)	$C_{24}(x,y)$	(7.42, 22.83)
$C_{13}(x,y)$	(1.79, 32.53)	α_1	105.88°
$C_{14}(x,y)$	(9.62, -2.74)	α_2	18.00°
$C_{20}(x,y)$	(12.70, 58.13)	w	0.80
$C_{21}(x,y)$	(35.23, 58.13)		

The out-of-plane thickness of the mechanism is set to t = 5 mm.

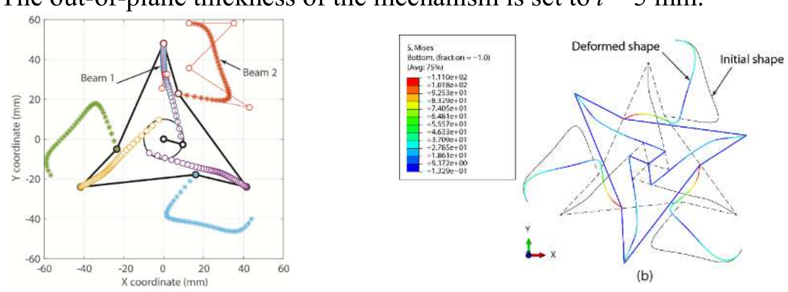
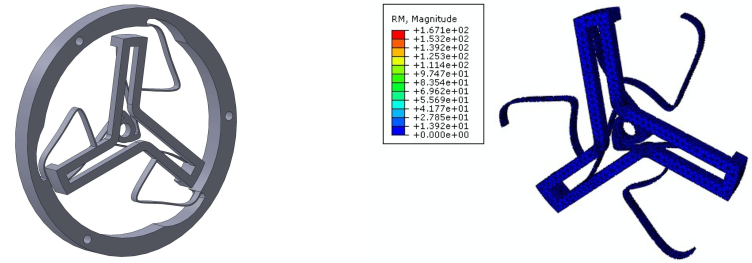


Fig 3.10 FEM beam element model (a) and simulation results (b)

The constant-torque range is divided into two stages at an input rotation angle of 58°. The torque deviation over the entire operating range from 30° to 110° is 3.8% and 4.3% for the 1D and 3D models, respectively.



(a) (b)

Fig 3.11 3D model of the CTM (a) and simulation results (b)

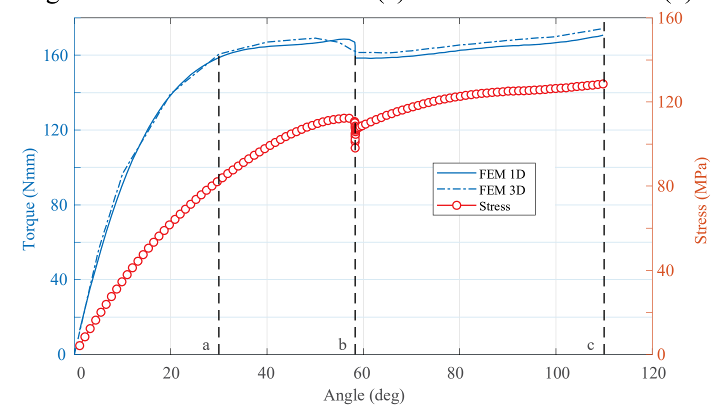


Fig 3.12 Torque and stress – rotation results of the CCTM-2

Table 3.5 Output torque deviation

Model	Average value (at 40°)	Minimum (at 30°)	Maximum (at 110°)	Maximum Deviation (%)
1D	164.5	158.6	170.8	3.8
3D	167.1	160.5	174.3	4.3

3.2 Optimization of CCTMs using NSGA-II and CBCM

3.2.1 Design and optimization of CCTM-3

3.2.1.1 Design Concept

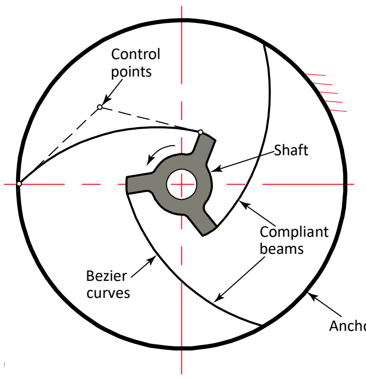


Fig 3.13 Concept of large-stroke CCTM

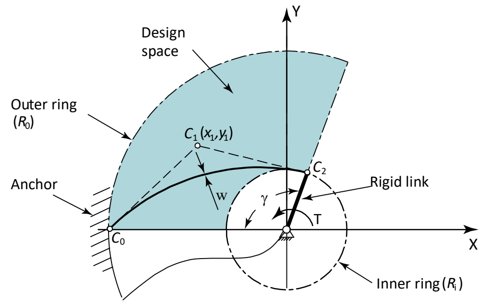


Fig 3.14 Design variables of the Bezier curve

3.2.1.2 Model analysis using CBCM

For a given input rotation angle, the resulting reaction torque is expressed as follows:

$$T = F_o R_i cos(\theta - \varphi_{20}) - P_o R_i sin(\theta - \varphi_{20}) + M_o \quad (3.16)$$

3.2.1.3 Optimization formulation

Two objective functions are considered in the optimization problem: 1) maximizing the working stroke S within the constant-torque region, as defined in Eq. (3.22); 2) minimizing the deviation between the output torque and the desired torque, as defined in Eq. (3.23).

$$Maximize(S) \quad (3.22)$$

với $S = b - a$

$$Minimize f(\theta) = \int_a^b (T - T_e)^2 \, d\theta \quad (3.23)$$

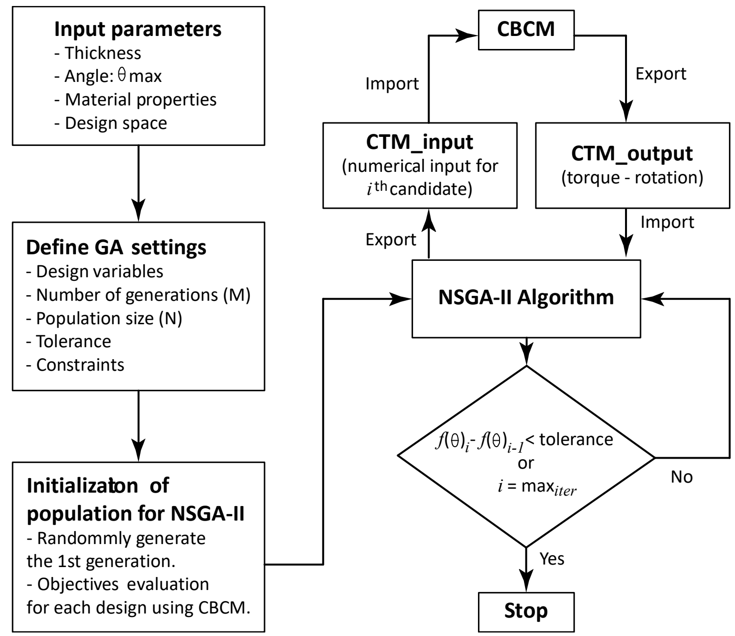


Fig 3.17 Flowchart of the CBCM-based NSGA-II optimization procedure

Table 3.6 Formulation of a large-stroke CCTM optimization

1. Objective functions: – Maximize the constant-torque operating range according to Eq. (3.22) – Minimize the variation of the torque according to Eq. (3.23)
2. Design variables: – Control points: $C_1(x_1, y_1)$ – In-plane thickness: w – Radii: R_o, R_i – Angle: γ
3. Constraints: i. g_1: $10 \leq R_i < R_o \leq 50$ (mm) ii. g_2: $R_i^2 \leq x_1^2 + y_1^2 \leq R_o^2$ iii. g_3: $\frac{\pi}{6} \leq \gamma \leq \frac{2\pi}{3}$ (rad) iv. g_4: $0.5 \leq w \leq 1.0$ (mm) v. g_5: $b - a > 65$ (deg) vi. g_6: $190 < T_e < 210$(Nmm) vii. Maximum stress within the CCTM, g_7: $\sigma_m < \sigma_y/SF$

PEEK is selected as the material for the mechanism, with an elastic modulus $E = 3.58\ GPa$ and a Poisson's ratio $\nu_p = 0.3$.

3.2.2 Results and discussion

3.2.2.1 Optimization results

The proposed CCTM achieves a wide and good constant-torque range of 80°, spanning from 5° to 85°.

Table 3.7 Optimal values of the design variables of CCTM-3

DVs	Values	Unit	DVs	Values	Unit
$C_1(x, y)$	(-30.9, 1.9)	mm	w	0.66	mm
R_i	16.4	mm	γ	110	deg
R_o	48.8	mm			

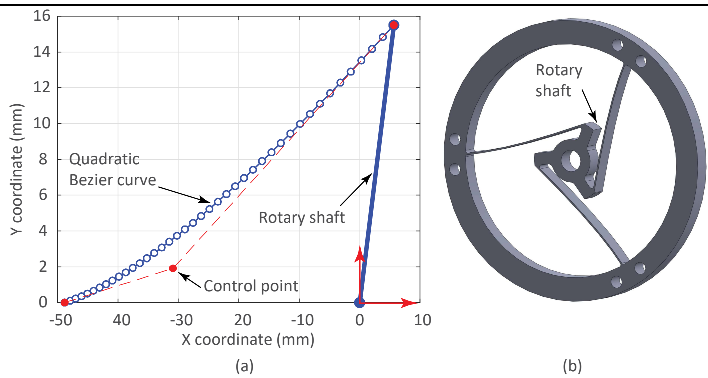
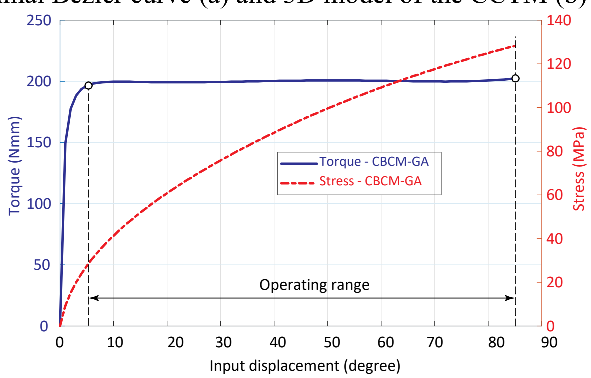


Fig 3.19 Optimal Bézier curve (a) and 3D model of the CCTM (b)



Hình 3.20 Results from CBCM-based NSGA-II optimization

3.2.2.2 Validation using FEM and experiments

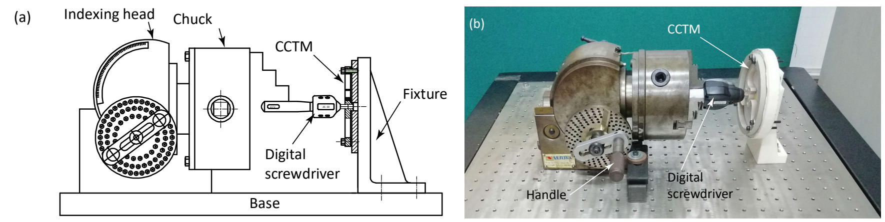


Fig 3.23 Experimental set-up: (a) design (b) actual testing model

The torque deviations within the operating range are 1.5%, 0.7%, and 3.7% for the CBCM model, FEM simulation, and experimental prototype, respectively, as summarized in Table 3.8. In addition, with a relatively small preload range of 5°, the proportion of the constant-torque region reaches 94.1%.

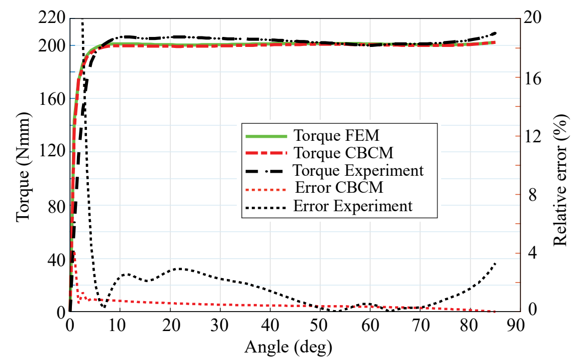


Fig 3.26 Comparison of the results from CBCM, FEA, and prototype

Table 3.8 Output torque deviation

Model	Average value	Minimum (at 5°)	Maximum (at 85°)	Maximum deviation (%)
CBCM	199.35	196.4	202.3	1.5
FEA	198.3	196.9	199.7	0.7
Experiment	201.5	194.0	209.0	3.7

The parameter S^*, defined as the ratio of the working stroke to the overall size of the mechanism, is employed as a criterion for evaluating compactness [24, 107]:

$$S^* = \frac{S}{D_o} \tag{3.24}$$

where S is the stroke, $D_o = 2R_o$, and R_o is illustrated in Fig 3.14.

As shown in Table 3.9, the proposed CCTM-3 achieves the largest stroke S and the highest value of S^*. These results indicate that the proposed design successfully fulfills the objective of achieving a larger constant-torque range while maintaining a compact structure.

Table 3.9 Comparison of the proposed CCTM with previous designs

Studies	S (degree)	D_o (mm)	S^* (mm/độ)
Bilancia [74]	40	100	0.40
Qiu [71]	26	60	0.43
Prakashah [32]	40	80	0.50
Hou [8]	50	90	0.56
Gandhi [33]	60	80	0.75
CCTM-3 [7]	**80**	**97.6**	**0.82**

3.2.3 Preliminary fatigue investigation of the CCTM

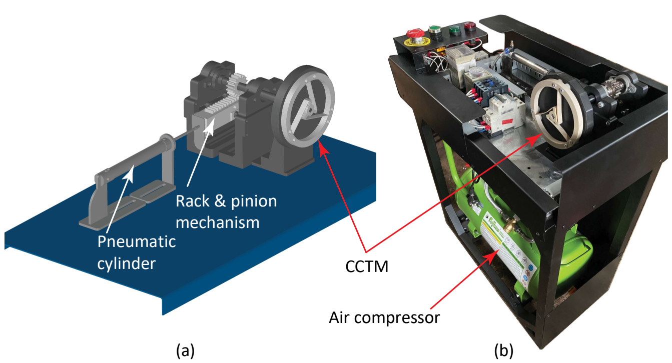
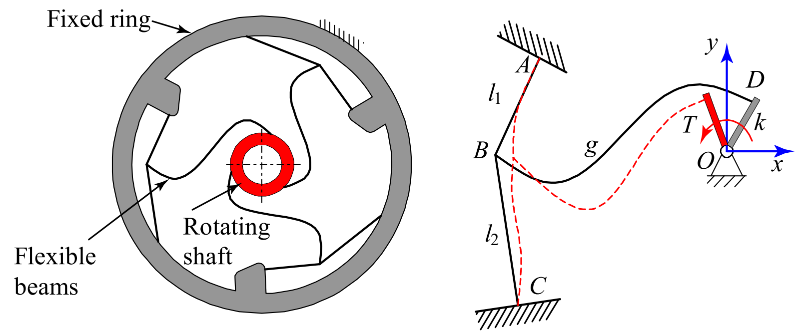


Fig 3.27 Fatigue testing machine for the proposed CCTM: (a) CAD design; (b) practical testing equipment

CHAPTER 4 RELIABILITY-BASED DESIGN AND OPTIMIZATION OF COMPLIANT CONSTANT-TORQUE MECHANISMS

4.1 Design and optimization of CCTM-4

4.1.1 Design concept



(a) (b)

Fig 4.1 Design concept of CCTM-4: (a) complete structure and (b) a single branch

4.1.2 Model analysis using CBCM

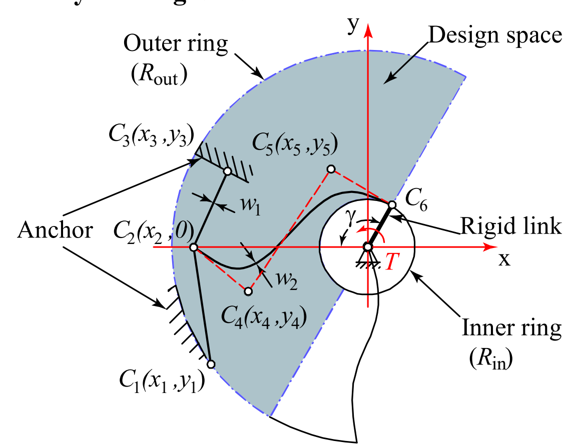


Fig 4.2 Design parameters of the CCTM-4

The reaction torque generated under an applied input rotation angle is determined by the following expression:

$$T = F_o R_{in} cos(\theta - \alpha_1 - \alpha_2 - \varphi_{19}) - P_o R_i sin(\theta - \alpha_1 - \alpha_2 - \varphi_{19}) + M_o \quad (4.20)$$

4.1.3 Reliability-based optimization of the mechanism

The two objective functions considered in this optimization problem are identical to those used for CCTM-3, with the working stroke defined as:

$$Maximize(S) \quad (4.34)$$

with $S = b - a$

$$Minimize f(\theta) = \int_a^b (T - T_e)^2 \, d\theta \qquad (4.35)$$

In addition to the objective functions and constraint functions introduced in conventional optimization, this optimization problem incorporates additional limit-state functions (*) for the reliability analysis stage.

Table 4.1 Formulation of the CCTM reliability-based optimization

1. Objective functions:
– Maximize the constant torque stroke according to Eq. (4.34)
– Minimize the output torque deviation according to Eq. (4.35)

2. Limit state function (*):
– $\{G(X_i) - D_0 \leq 0\} \geq \Phi(\beta)$ (4.36)
– With eight random variables: $\{X_i\} = \{x_2, y_2, x_4, y_4, x_5, y_5, w_1, w_2\}$
– Reliability $R \geq 99\%$

3. Design variables:
– Coordinates of control points: $x_1, y_1, x_2, x_3, y_3, x_4, y_4, x_5, y_5$
– Beam widths: w_1, w_2
– Inner ring's radius: R_{in}
– Initial angle of the rotating shaft: γ

4. Constraints:
i. g_1: $8 \leq R_{in} \leq 12$ (mm)
ii. g_2: $R_{in}^2 \leq x_i^2 + y_i^2 \leq R_{out}^2$ $(R_{out} \leq 50;\ i = 1,2,\ldots,5)$
iii. g_3: $x_2 < x_4 < x_5$
iv. g_4: $y_1 < 0 < y_3$
v. g_5: $\frac{\pi}{4} \leq \gamma \leq \frac{3\pi}{4}$ (rad)
vi. g_6: $0.5 \leq w_i \leq 1.0$ (mm) $(i = 1,2)$
vii. g_7: $230 < T_e < 260$(Nmm)
viii. Maximum stress, g_8: $\sigma_m < \sigma_y/SF$
ix. Constant-torque stroke (degree): (*)
g_9: $S - 70 \leq 0$
x. Output torque deviation: (*)
g_{10}: $|T - T_e| - 0.03T_e \leq 0$

The random variables considered in this reliability-based optimization include the coordinates $x_2, y_2, x_4, y_4, x_5, y_5$, and the beam widths w_1, w_2. The standard deviation (STD) of these variables is set to 0.05 mm.

The First-Order Reliability Method (FORM) is adopted to evaluate reliability during the optimization process. PEEK is selected as the material for

the mechanism, with an elastic modulus $E = 3.58\ GPa$ and a Poisson's ratio $\nu_p = 0.3$. The thickness of the mechanism is fixed at 10.0 mm.

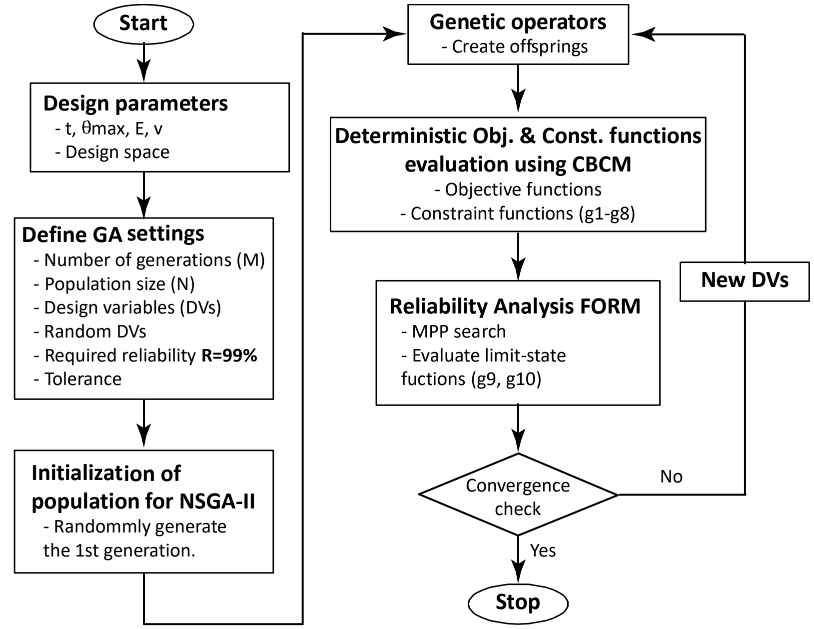


Fig 4.8 Flowchart of the reliability-based optimization algorithm

4.2 Optimization results

The results of the reliability-based optimization are summarized in Table 4.2, yielding a reliability index of $\beta = 3.0281$, which corresponds to a reliability level of 99.88%. The optimized mechanism achieves a wide constant-torque range of 88°, spanning from 17^o to 105^o, with a torque deviation of 2.8% relative to the desired torque. Therefore, the mechanism satisfies the output torque error requirement of 3%.

Table 4.2 Optimal values of the design variables of CCTM-4

DVs	Value	Unit	DVs	Value	Unit
(x_1, y_1)	(-39.88, -15.42)	mm	w_1	0.53	mm
x_2	-38.05	mm	w_2	0.60	mm
(x_3, y_3)	(-30.58, 16.45)	mm	R_{in}	10.50	mm
(x_4, y_4)	(-30.64, -14.36)	mm	γ	120	độ
(x_5, y_5)	(-8.63, 19.88)	mm			

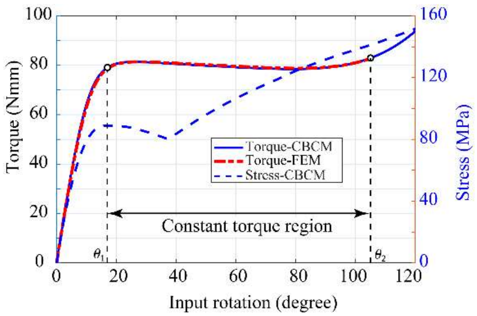


Fig 4.9 Torque and stress results obtained from reliability-based optimization

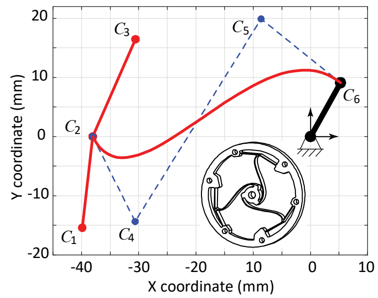


Fig 4.11 Optimization results of a single branch and the 3D model of the mechanism

4.3 Validation using FEM and experiments

To conduct experimental torque measurements, a CCTM prototype fabricated from PEEK was manufactured using CNC milling. The fabricated mechanism was subsequently 3D-scanned to compare the actual geometry with the designed model. The results indicate that the average beam widths satisfy the design requirements. Overall, the standard deviations of the positions of all three curves remain within the allowable STD of 0.05 mm.

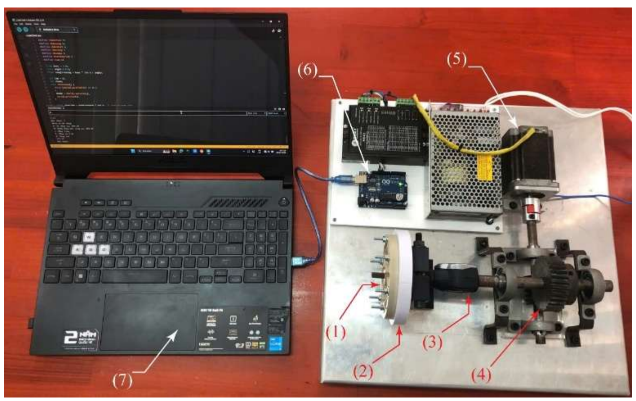


Fig 4.14 Experimental setup for torque measurement

The maximum deviation of the experimental torque is 1.67%, occurring at an input rotation angle of 20°. The average experimental torque over the range from 17° to 105° is 240.4 N·mm. The maximum deviation of 2.9% between the average experimental torque and the desired torque occurs at an input angle of 30°, where the torque value reaches 247.39 N·mm. With deviations below 3%, the fabricated mechanism remains within the allowable error range and satisfies the specified design requirements.

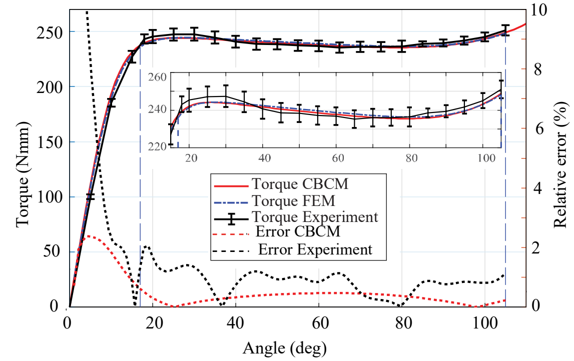


Fig 4.16 Comparison of torque results obtained from CBCM, FEM, and experiments

As shown in Figure 4.18, the proposed design successfully fulfills the intended design objectives.

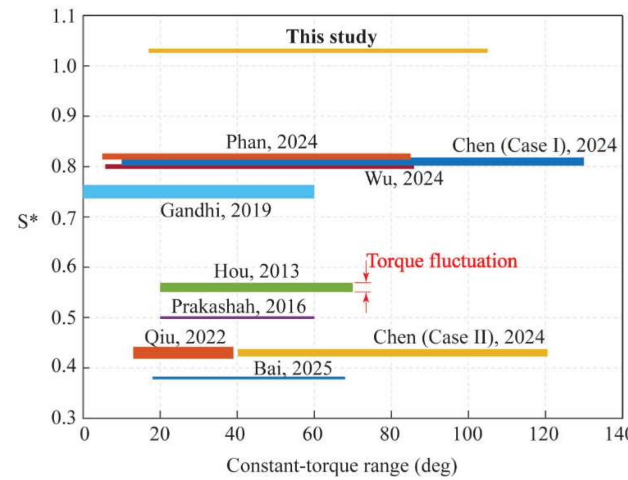


Fig 4.18 Comparison of CCTM-4 with previously published CCTMs

CHAPTER 5 PRACTICAL APPLICATIONS OF COMPLIANT CONSTANT-TORQUE MECHANISMS

5.1 Requirements for biomechanical device applications

The design and application of compliant mechanisms in the field of biomechanics require strict compliance with technical standards to ensure biological safety, mechanical reliability, and safe interaction between the device and the human body.

5.2 Knee joint assistive device

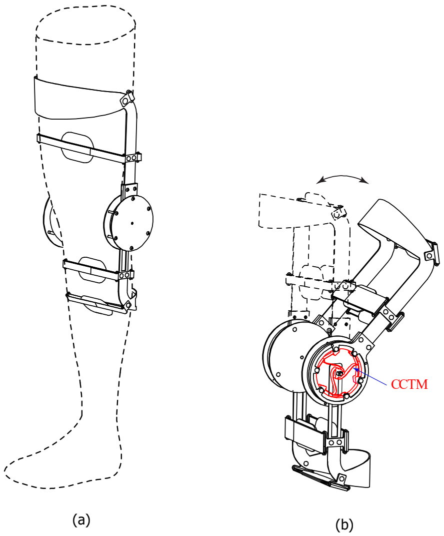


Fig 5.1 Knee joint assistive device integrated with a CCTM: (a) device worn on the human leg and (b) operating principle

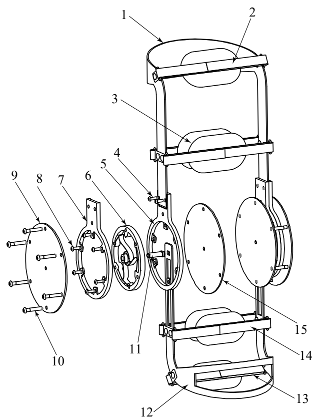


Fig 5.2 Structure of the knee joint assistive device integrated with a CCTM

CHAPTER 6 CONCLUSIONS AND FUTURE WORK

6.1 Conclusions

This dissertation presents several novel contributions of both methodological and practical significance in the design of compliant constant-torque mechanisms (CCTMs).

First, the dissertation develops three automated synthesis and optimization frameworks for CCTMs, fully implemented in MATLAB. Through these frameworks, four novel CCTM designs are synthesized, achieving significant improvements in output torque accuracy, constant-torque operating range, and structural compactness compared with existing CCTM studies.

- **The first framework** employs a genetic algorithm (GA) combined with finite element analysis (FEM) using ABAQUS to synthesize two new CCTM designs. Among them, CCTM-2 achieves an operating range of up to 110°, with a constant-torque region spanning from 30° to 110°. Notably, CCTM-2 introduces a novel design approach based on stacking compliant beams in layers with sequential deformation, thereby extending the working stroke while ensuring that material elastic limits are not exceeded.
- **The second framework** constructs a mathematical model of the CCTM based on the CBCM to compute displacement, torque, and stress, and subsequently integrates this model into the multi-objective genetic algorithm NSGA-II. This approach significantly improves optimization efficiency and reduces computational time, resulting in the CCTM-3 design, which features a compact structure while achieving a constant-torque stroke of up to 80°. Remarkably, the constant-torque ratio of CCTM-3 reaches 94.1% over the entire operating range, with an approximately constant torque of 200 N·mm. The output torque deviation is about 1.5% for theoretical and numerical models and 3.7% when compared with experimental results.
- **The third framework** integrates CBCM, First-Order Reliability Method (FORM), and NSGA-II to perform reliability-based optimization of CCTMs. In this framework, random factors affecting the geometry and dimensions of compliant beams due to manufacturing errors are explicitly considered to ensure that the fabricated mechanisms consistently meet design objectives. The resulting design, CCTM-4, exhibits an extremely compact configuration achieved by introducing a flexure hinge at one end of the curved beam to replace the traditional fixed joint, thereby reducing stress concentration and overall mechanism size. CCTM-4 achieves a constant-torque range of up to 88°, with output torque deviation below 3% at a reliability level of 99.88%. These results confirm the validity and strong potential of reliability-based optimization for CCTM design.

Second, through the proposed CCTM synthesis frameworks, the dissertation demonstrates a novel and effective application of the CBCM, a powerful and accurate tool for analyzing large deformations of compliant beams with arbitrary curved geometries. The integration of CBCM into the multi-objective optimization algorithm NSGA-II significantly reduces computational time compared with conventional finite element methods, thereby enhancing optimization efficiency and enabling rapid and accurate identification of optimal CCTM designs.

Third, the dissertation successfully incorporates FORM-based reliability analysis into the CBCM-driven multi-objective NSGA-II optimization framework, forming a reliability-based design optimization (RBDO) problem. This approach enables the identification of optimal designs with high reliability, ensuring that fabricated mechanisms continue to satisfy design requirements despite random manufacturing-induced uncertainties. This reliability-based optimization framework opens new research directions for compliant mechanisms in general and CCTMs in particular.

Fourth, the dissertation establishes experimental platforms to support the evaluation and further development of CCTMs. Specifically, a cyclic loading test system is designed and fabricated to preliminarily investigate the working behavior of CCTMs under repetitive loading. In addition, a method for evaluating post-machining geometric and dimensional errors of arbitrarily curved compliant beams is proposed, contributing to improved manufacturing accuracy for compliant mechanisms with complex geometries.

Fifth, from an application perspective, the dissertation proposes a knee joint assistive device integrating a CCTM to enhance mobility for individuals with impaired knee function. Although the work is currently at the conceptual stage, analytical results indicate that CCTMs offer advantages over conventional springs by providing more favorable assistance characteristics, thereby facilitating daily activities such as walking and stair climbing. Consequently, this concept has been registered for patent protection and is being further developed.

6.2 Future work

- Further miniaturization of CCTMs to enhance practical applicability, along with the exploration of alternative materials or methods to increase output torque, targeting assistive joint devices requiring high torque levels, such as knee joints.
- Investigation of manufacturing methods for CCTMs to achieve higher accuracy and improved cost efficiency.
- Development of in-depth studies on the fatigue behavior of CCTMs, together with the proposal of stress-reduction strategies and lifetime enhancement methods, particularly under repetitive loading conditions.